represented uniquely by paths, which are made up of the sides of the net and whose initial points coincide with the same vertex of N, arbitrarily chosen. The terminal points of the paths representing two equal finite products coincide with the same vertex of N.

We can associate with the net N an infinite two-manifold which can be mapped topologically on the interior E_2 of a circle S. The points of S may be taken to represent the reduced infinite products and thus the infinite paths of the transformed net representing these products, and will be called the *ideal elements of the infinite two-manifold.* By a proper definition of continuity on $E_2 + S$, we can prove *that the infinite two-manifold and its ideal elements defined by means of the group is a closed two-cell.*

[1] Research Fellow of China Foundation. The author wishes to thank Professor S. Lefschetz for valuable suggestions and encouragement in connection with this investigation.

[2] Nielson, J., *Acta Mathematica*, **50,** 189–379 (1927). This is the third of his four papers on this subject.

[3] We agree to set $(G_m) \equiv (G_n)$ and $(G_m)' \equiv (G_n)'$, if $m = cp + n$, c being an integer and $1 \leqq n \leqq p$.

INVERSE COMMUTATOR SUBGROUPS

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If s and t represent any two operators of a given group G then the operator $s^{-1}t^{-1}st$ is commonly called the commutator of s and t. In the present article the operator $s^{-1}t^{-1}st^{-1}$ is defined as the *inverse commutator* of s and t and some fundamental properties of these commutators are developed. Since the transform of an inverse commutator by any operator of the group is an inverse commutator of this group it results directly that all the inverse commutators of a group generate an invariant subgroup of this group which will be called the *inverse commutator subgroup.* The corresponding quotient group cannot involve any operator whose order exceeds 2 and hence it must be the abelian group of order 2^m, and of type $(1, 1, 1, \ldots)$. Since all the inverse commutators of such a group are obviously equal to the identity it results directly that *the inverse commutators of a group generate its smallest invariant subgroup which gives rise to an abelian quotient group of order 2^m and of type $(1, 1, 1, \ldots)$, and every invariant subgroup which gives rise to such an abelian quotient group must involve the inverse commutator subgroup.*

From this theorem it results directly that the inverse commutator subgroup of every group whose order is of the form 2^m is its ϕ-subgroup and if the index of this subgroup is 2^α then the number of operators in every set of independent generators of the group is exactly α, and vice versa. A necessary and sufficient condition that the commutator of s and t is the identity is that s and t are commutative while a necessary and sufficient condition that the inverse commutator of s and t is the identity is that s transforms t into its inverse. It should be noted that when s transforms t into its inverse it is not necessarily true that t also transforms s into its inverse. The concept of inverse commutator seems to be especially useful when t is supposed to represent successively all the operators of an abelian group H which is transformed according to an automorphism of order 2 by s. Hence we shall assume in what follows that s and t satisfy these conditions so that only special inverse commutators and special inverse commutator subgroups will be under consideration unless the contrary is stated.

Since H is abelian all of its operators which correspond to their inverses under an automorphism of H must constitute a subgroup of H. The inverse commutator subgroup which corresponds to an automorphism of H is therefore simply isomorphic with the quotient group of H with respect to the subgroup formed by all the operators of H which correspond to their inverses under this automorphism. In particular, when this automorphism is of order 2 the corresponding inverse commutator subgroup must be composed of operators which correspond to themselves under this automorphism. It therefore results that when H is of odd order its operators which correspond to their inverses under an automorphism of order 2 constitute a subgroup which has only the identity in common with the inverse commutator subgroup resulting from this automorphism. The concept of inverse commutator subgroup therefore furnishes a direct proof of the theorem that every abelian group of odd order is the direct product of the two subgroups composed respectively of its operators which correspond to themselves and to their inverses under an automorphism of order 2.[1]

If H is a cyclic group of order p^m, p being a prime number, it results directly from the preceding paragraph that its group of isomorphisms involves only one operator of order 2 when $p > 2$. When $p = 2$ and $m > 2$ it results from similar considerations that this group of isomorphisms involves three and only three operators of order 2. In fact, if the order of the inverse commutator subgroup in an automorphism of order 2 of this group exceeds 2 it must be 2^{m-1} since the operators of order 4 must then correspond to themselves. As all the operators of this commutator subgroup must also correspond to themselves under this automorphism it results that either all the operators correspond to themselves or that only the operators of highest order are multiplied by the operator of order 2 when the inverse commutator subgroup corresponding to an automorphism of order 2 of this group is of order 2^{m-1}. There is obviously one such automorphism of order 2 when the inverse commutator subgroup is the identity and one when this subgroup is of order 2.

As a further illustration of the use of these special inverse commutator subgroups, and on account of the results, we shall employ them to determine all the groups which have the property that they involve a given abelian subgroup H of index 2 while all the additional operators are of order 4. Since $stst = s^2s^{-1}tst = s^2t_0$, where t_0 is an inverse commutator of G, and since t_0 is commutative with s, it results that t_0 is either the identity or of order 2. It cannot be equal to s^2 since st is assumed to be of order 4. This proves the following theorem: *If a group contains an abelian subgroup H of index 2 and all of its remaining operators are of order 4 then its inverse commutators constitute a subgroup involving no square of an operator not found in H. If this subgroup is extended by such a square there results a subgroup of the central which involves no operator whose order exceeds 2 and only such squares in addition to the inverse commutator subgroup.*

On the other hand it is easy to prove that when an abelian group H contains a subgroup K involving no operator whose order exceeds 2 and K is contained in a subgroup of H corresponding to a quotient group which is isomorphic to a subgroup of index 2 in K, then H can be extended to a group G of twice the order of H and such that each of the additional operators is of order 4. The number of the different squares of these additional operators is always equal to the order of the inverse commutator subgroup of H under G. In particular, a necessary and sufficient condition that all these additional operators have the same square is that s transforms every operator of H into its inverse. If such a G can be constructed so that it has 2^{α}, $\alpha > 0$, such distinct squares then that it is always possible to construct with the same H a G which involves any lower power of 2 such distinct squares. The theorem noted at the close of the preceding paragraph involves therefore a necessary and sufficient condition that an abelian group may be extended to a group of twice its order and such that each of the added operators is of order 4.

A necessary and sufficient condition that the inverse commutator subgroup corresponding to an automorphism of G is always identical with the commutator subgroup corresponding to the same automorphism is that G is abelian, of order 2^m, and of type $(1, 1, 1, \ldots)$. Hence it results directly that the commutators corresponding to an automorphism of order 2 of such a group must correspond to themselves under this automorphism. The special inverse commutator subgroup in which t is an operator of an abelian group H which is transformed into itself by s and involves s^2 is composed of the smallest subgroup of G which gives rise to a quotient group which is either dihedral or generalized dihedral. Hence it follows that the concepts of inverse commutator subgroup and special inverse commutator subgroup enable us to unify a number of fundamental theorems of groups of finite order. This unification is the main object of the present note. While the identity automorphism gives rise only to the identity commutator it gives rise to the squares of all the operators of the group as inverse commutators.

[1] G. A. Miller, *Trans. Am. Math. Soc.*, **10**, 472 (1909).

THE CONVENTION OF EQUIDIMENSIONAL ELECTRIC AND MAGNETIC UNITS

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It is the object of this paper to present a set of electromagnetic unit dimensions, common to both electric and magnetic systems, based on the conventional hypothesis that the dimensions of permittivity κ_0 and permeability μ_0 for free space are the same.

It was announced by Maxwell in 1881 in his Treatise,[1] Vol. 2, Chap. X, that "every electromagnetic quantity may be defined with reference to the fundamental units of Length, Mass, and Time." He formulated the "dimensions" of each quantity, such as Resistance, Current, Capacitance, etc., in terms of the fundamental quantities of dynamics—L, M, and T. Thus with any velocity V, defined as a ratio L/T, the dimensions of velocity would be L^1, M^0, T^{-1}, or giving only the exponents, as (1, 0, -1). Similarly, the ordinary dynamic formulas of energy W, and of power P, being respectively $MV^2/2$ and W/T, their exponential dimensional formulas would be (2, 1, -2) and (2, 1, -3).

Maxwell also showed that there were always two different dimensional formulas for each electromagnetic quantity: namely, one in the electric (electrostatic) system, and one in the magnetic (electromagnetic) system. The electric units were derived from the force of repulsion between like electric charges across a known distance, assuming that the permittivity κ_0 of free space is the numeric unity. The magnetic units were, however, derived from the force of repulsion between like magnetic poles across a known distance, assuming that the permeability μ_0 of free space is the numeric unity.

The accompanying table 1 is based on Maxwell's list of dimensions for